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BARBELL GRAPHS ON b-COLORING

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Abstract

φ(G) is the b-chromatic number of G, is the maximum k for which G has a b-coloring by k colors. A b-coloring of G by k colors is proper k-coloring of the vertices of G such that in each color class i there exists a vertex x_i having neighbors in all the other k-1 color classes. Such avertex x_i is called a b-dominating vertex, and the set of vertices $\{x_1,x_2,\ldots x_k\}$is called a b-dominating system. In this paper, we investigate the b-coloring of corona graph of barbell graph with cycle graph, cycle graph with barbell graph denoted by, $B(K_{n,}K_n)$ °C_n C_n°$B(K_{n,}K_n)$ respectively.

Keywords: b-coloring, b-chromatic Number, Barbell Graph, Path Graph, Cycle Graph.

1. Introduction

Graph theory is the theory of graphs dealing with nodes and connections or vertices and edges. The subject has skilled explosive growth, due in large measure to its role as a necessary arrangement underpinning modern applied mathematics. Configurations of nodes and connections have great diversity of applications. They may be physical networks, such as electrical circuits, roadways, or organic molecules. Let G,V be a graph with vertex set and edge set E.

A k-coloring of a graph G is a partition P $=\{v_1,v_2,v_3,\ldots.v_k\}$of V into independent sets of G. The minimum cardinality k for which G has a k-coloring is the chromatic number $\chi\ (G)$of G. The b-chromatic number $\varphi(G)$of a graph G isthe largest positive integer k such that G admits a properk-coloring in which every color class has a representative neighboring to at least one vertex in each of the other color classes. Such a coloring is called a b-coloring.

The b-chromatic number was introduced by Irving and Manlove [1] considering proper colorings that are minimal with respect to a partial order defined on the set of all partitions of V(G). They have shown that determination of$\varphi(G)$ is NP-hard for generalgraphs, but polynomial for trees. There has been an increasinginterest in the study of b-coloring since the publication of [1]. Kouider and Maheo [5] gave some lower and upper bounds for the b-chromatic number of the cartesian product of two graphs. Kratochvil [6] characterized bipartite graphs for which the lower bound on the b-chromatic number is attained and proved the NP-completeness of the problem to decide whether there is a connected bipartite graphs which is dominating proper b-coloring even for with k $=\Delta(G)+1$. Corteel [7] proved that the b-chromatic number problem is not approximable within 120/133 -∈ for any∈> 0, unless P = NP. Lot of interesting b-colouring research works done by Vernold Vivin and Venkatachalam [9], Vijayalakshmi and M.Kalpana[11] .

2. Preliminaries

The G_1 and G_2 are two graphs, the corona of two graph is G= G_1^{O} G2 formed from one copy of G_1 and |V(G1)| copies of G_2 where the i^{th} vertex of G_1 and every vertex in the i^{th}copy ofG_2 are adjacent. This kind of product was introduced by Harary and Frucht in 1970 [3]. The barbell graph [10] is constructed by connecting two arbitrary connected graph s and *H* by a bridge.

3. Main Results

The b-coloring of corona graph of Barbell graph with Cycle graph Algorithm:3.1.

Input: $B(K_{n,}\ K_n)$ °C_n , n≥3.

V $\leftarrow$ {$x_1,x_2,x_3,\ldots,x_n,y_1,y_2,y_3\ldots\ldots\ldots y_n,\ p_1^1,\ p_2^1,\ldots\ p_n^n, q_1^1, q_2^1,\ldots\ldots q_n^n$}.

for i=1 to n

$x_i \leftarrow i$;

end for

for i=1 to n

$y_i \leftarrow n+i$;

end for

for i=1 to n, j=1 to n

$p_j^i \leftarrow n+j$;

end for

for i=1 to n, j=1 to n

$q_j^i \leftarrow j$;

end for

end procedure

output: vertex colored $B(K_n, K_n)\circ C_n$.

Theorem:3.1

For any barbell graph $B(K_n, K_n)$ and cycle graph C_n,the b-chromatic number of corona graph of $B(K_n° K_n)\circ C_n$ is

$$\varphi\,[\,B(K_n,K_n)\circ C_n]=2n,\ n\geq 3.$$

Proof:

Let $V[B(K_n,K_n)] = \{x_i:1\leq i\leq n\}\cup\{y_i:1\leq i\leq n\}$ and

$$V(C_n) = \{p_i:1\leq i\leq n\}$$

By the definition of corona product, each vertex of $B(K_n° K_n)$ is adjacent to every vertex of number of copies of C_n.

ie., $\{x_i,y_i: 1\leq i\leq n\}\in V(B(K_n,K_n))$, is adjacent to every vertex of

$\{p_j^i, q_j^i : 1\leq i\leq n, 1\leq j\leq n\}\in V(C_n)$

Then the vertex set of the corona product $B(K_n,K_n)\circ C_n$ is

$V[B(K_n, K_n)\circ C_n]= \{x_i: 1\leq i\leq n\} \cup \{y_i: 1\leq i\leq n\}$

$\cup\ \{p_j^i: 1\leq i\leq n, 1\leq j\leq n\}\cup\ \{q_j^i: 1\leq i\leq n, 1\leq j\leq n\}$

Assign the coloring as per the algorithm 3.2

By this coloring procedure, we get that

$\varphi\,[\,B(K_n,K_n)\circ C_n]\geq 2n$

To prove the lower bound, let us assume that, the b-chromatic number of corona product of barbell graph $B(K_n, K_n)$ with cycle graph C_n is greater than 2n.

That is the b-chromatic number of $B(K_n, K_n)\circ C_n$ is equal to 2n+1.

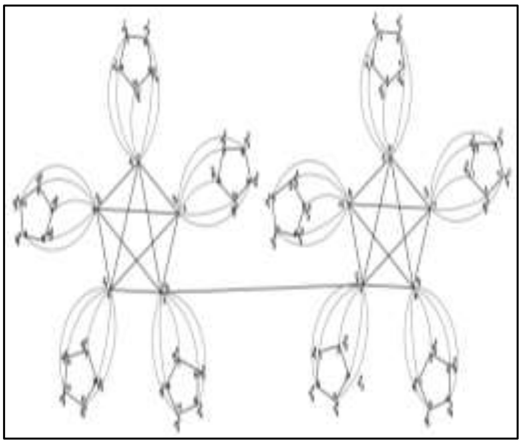
Assigning 2n+1 colors, the graph $B(K_n, K_n)\circ C_n$ should have 2n+1 vertices with 2n degree along with distinct colors. But the graph

$B(K_n, K_n)\circ C_n$ having only 2n vertices with maximum degree 2n-1, which is the contradiction.

Therefore assigning 2n+1 colors is impossible.

$\therefore \varphi\,[\,B(K_n,K_n)\circ C_n]\ \leq\ 2n$

Hence $\varphi\,[B(K_n,K_n)\circ C_n] = 2n$.



$\varphi[B(K_5, K_5)°C_5] = 10$

The b-coloring of corona graph of Cycle graph with Barbell graph Algorithm:3.2

Input: $C_n°B(K_n,K_n)$, $n \geq 3$.

$V \leftarrow \{a_1, a_2, \ldots, a_n, x_1^1, x_2^1, \ldots, x_n^n, y_1^1, y_2^1, \ldots, y_n^n\}$.

for i=1 to n
$a_i \leftarrow i$;
end for
for i=1 to n, j=1 to n-1, k=1 to n
$x_j^i \leftarrow k$;
end for
for j=n
$x_j^i \leftarrow n+1$;
end for
for i=1 to n, j=1 to n-1, k=1 to n
$y_j^i \leftarrow k$;
end for
for j=n
$y_j^i \leftarrow n+2$;
end for
end procedure
output: vertex colored $C_n°B(K_n, K_n)$.

Theorem:3.2

For any cycle graph C_n and barbell graph $B(K_n, K_n)$, the b-chromatic number is
$\varphi[C_n°B(K_n,K_n)] = n+2$, $n \geq 3$.

Proof:

Let $V(C_n) = \{a_i : 1 \leq i \leq n\}$ and

$V[B(K_n,K_n)] = \{x_i : 1 \leq i \leq n\} \cup \{y_i : 1 \leq i \leq n\}$

By the definition of corona product each vertex of C_n is adjacent to every vertex of number of copies of $B(K_n, K_n)$, then the vertex set of the corona product $C_n°B(K_n, K_n)$ is

$V[C_n°B(K_n,K_n)] = \{a_i : 1 \leq i \leq n\} \cup \{x_j^i : 1 \leq i \leq n, 1 \leq j \leq n\}$
$\cup \{y_j^i : 1 \leq i \leq n, 1 \leq j \leq n\}$

Assign the colors as per the algorithm 3.3.
By this coloring procedure, we have that
$\varphi[C_n°B(K_n,K_n)] \geq n+2$

To prove the lower-bound, let us assume that, b-chromatic number of corona product of cycle graph C_n with barbell graph $B(K_n, K_n)$ is greater than n+2.

That is the b-chromatic number of $[C_n \circ B(K_n, K_n)] = n+3$

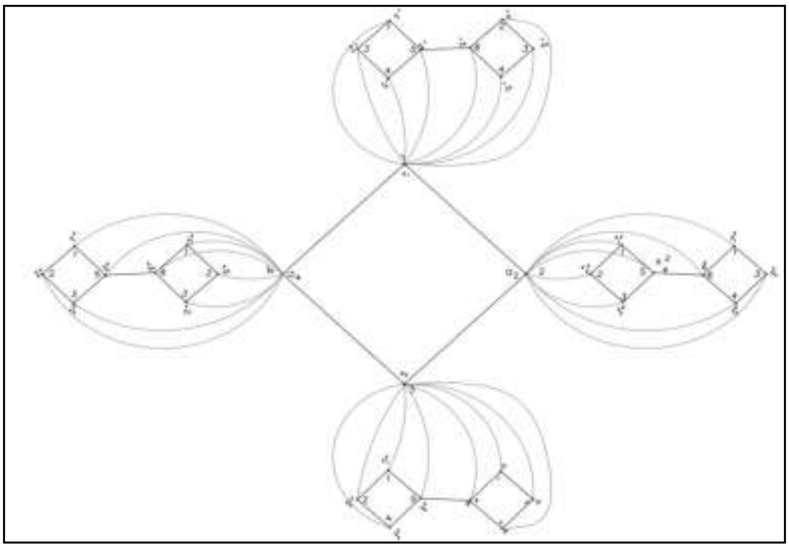
We can assign n+3 colors only if the graph having n+3 vertices assigned with n+3 distinct colors which are adjacent to each other.

Here the colour class c_{n+3} is not adjacent to the colour class c_{n+2}, c_{n+1}&c_n which is the contradiction.

Therefore assigning n+3 colours is not possible.

$\therefore \varphi[C_n \circ B(K_n, K_n)] \leq n+2$

Hence $\varphi[C_n \circ B(K_n, K_n)] = n+2$.



$C_4 \circ B(K_4, K_4) = 6$

References:

1. R.W. Irving, D.F. Manlove, The b-chromatic number of a graph, Discrete Appl. Math. 91 (1999), 127–141.
2. J.A.Bondy and U.S.R. Murty, Graph Theory with Applications. MacMillan, London, 1976
3. R. Frucht and F. Harary, On the corona of two graphs, A equations Math., 4(1970), 322.325.
4. Roberto Frucht and Frank Harary, On the Corona of Two Graphs, An equation Math.vol 4, 1970, 322-325.
5. M. Kouider, M. Maheo, Some bounds for the b-chromatic number of a graph, Discrete Math. 256 (1-2) (2002) 267–277.
6. J. Kratochvil, Z. Tuza, M. Voigt, On the b-chromatic number of graphs, 28th International Workshop on Graph-Theoretic Concepts in Computer Science, vol. 2573, Cesky Krumlov, Czech Republic, LNCS, 2002, pp. 310–320.
7. S. Corteel, M. Valencia-Pabon, J.C. Vera, on approximating the b-chromatic number, Discrete Appl. Math. 146 (2005) 106–110.
8. M. Jakovac, S. Klavzˇ ar, Theb-chromatic number of cubic graphs, Graphs Comb. 26 (1) (2010) 107–118.
9. M. Venkatachalam, J. Vernold Vivin, The b-chromatic number of star graph families, Le Matematiche 65 (1) (2010) 119–125.
10. Asmiati,I. Ketut Sadha Gunce Yanaand Lyra Yulianti. On the Locating Chromatic Number of Certain Barbell Graphs, International Journal of Mathematics and Mathematical Sciences Volume 2018, Article ID 5327504, 5 pages
11. M. Kalpana, D. Vijayalakshmi, on b-coloring of tadpole graphs ,journal of applied science and computations,5(10),(2018) 589-594.